purpose, the accuracy of the Mach-wave method was far superior to that of the pressure technique.

IV. Conclusions

The Mach-number distribution in the Mach 2.3 nozzle of the new Ludwieg tube at the California Institute of Technology has been measured using the Mach-wave intersection method. The centerline Mach-number distribution agrees very well with the design curve, and the flow in the test section is parallel to within the accuracy of the method. The Mach number in the test section is 2.30 ± 0.03. The Ludwieg tube principle is very well suited to university research because of the low cost of operation and the clean flow it provides.

Acknowledgments

We are very grateful to the Anna Wild Estate, which provided funding for the construction of the Ludwieg tube. We thank Nicolas Ponchaut and Bahram Valiferdowsi for their generous help with the experiments.

References

[1]Ludwieg, H., Hottner, T., and Grauer-Carstensen, H., "Der Rohrwindkanal der Aerodynamischen Versuchsanstalt Göttingen," *Jahrbuch 1969 der DGLR*, Deutsche Gesellschaft für Luft- und Raumfarht, 1969, pp. 52–58.

[2]Meyer, T., "Über zweidimensionale Bewegungsvorgänge in einem Gas, das mit Überschallgeschwindigkeit strömt," Ph.D. Dissertation, Georgia Augusta Univ., Göttingen, Germany, 1908.

[3]Whitham, G. B., *Linear and Nonlinear Waves*, Wiley, New York, 1974.

[4]Quirk, J. J., "Amrita—A Computational Facility (for CFD Modelling)," von Kármán Inst. for Fluid Dynamics, 29th CFD Lecture Series, von Kármán Inst. for Fluid Dynamics, Brussels, 1998.

M. Sichel
Associate Editor

Applicability of a Simple Method for Thermal Postbuckling of Square Plates

G. Venkateswara Rao* and K. Kanaka Raju†
Vikram Sarabhai Space Center,
Trivandrum 695 022, India

Nomenclature

a	=	length of the sides of the square plates
D	=	plate flexural rigidity, $Et^3/12(1 - \nu^2)$
E	=	Young's modulus
N	=	uniform biaxial compressive load developed in the square plate due to temperature rise of ΔT
N_{cr}	=	buckling load of the square plate under uniform biaxial compression
N_x, N_y	=	uniform loads on the square plate in the x and y directions
T	=	uniform biaxial tension developed in the square plate
T_x, T_y	=	uniform tensions developed in the square plate in the x and y directions
t	=	thickness of the square plate
u, v	=	in-plane displacements of the square plate in the x and y directions
w	=	lateral displacement
x, y	=	in-plane coordinates (Fig. 1)
α	=	coefficient of linear thermal expansion
β	=	as defined in Eq. (1)
ΔT	=	temperature rise from the stress free temperature of the square plate
δ	=	central deflection of the square plate
λ_L	=	thermal buckling load parameter defined as $N_{cr}a^2/\pi^2 D$
$\lambda_{Nx}, \lambda_{Ny}$	=	nondimensional parameters, defined as $N_x a^2/\pi^2 D$ and $N_y a^2/\pi^2 D$, respectively
λ_{PB}	=	thermal postbuckling parameter
λ_T	=	nondimensional uniform biaxial tension parameter, defined as $Ta^2/\pi^2 D$
$\lambda_{Tx}, \lambda_{Ty}$	=	nondimensional uniform tension parameters, defined as $T_x a^2/\pi^2 D$ and $T_y a^2/\pi^2 D$, respectively
ν	=	Poisson's ratio

Introduction

AEROSPACE structural elements such as uniform columns and plates are subjected to severe environmental conditions, including high thermal loads. The low margins of safety involved in their design lead the designers to use the postbuckling strength of the buckling-prone structural elements, wherever possible.

Classical solutions for predicting the postbuckling strength of these structural elements subjected to mechanical loads are presented by Thompson and Hunt[1] and Dym.[2] A similar study, in which these structural elements are subjected to thermal loads (uniform temperature rise from the stress-free condition of the plate), with ends not free to move axially in the case of columns or with restrained in-plane displacements normal to the edges in the case of the square plates (from now on called immovable edges), is presented by the authors,[3,4] using the versatile finite element method and verified by the classical Rayleigh–Ritz method for square plates with all sides simply supported. It is found from these studies that the thermal postbuckling strength of structural elements is an order of magnitude higher than the mechanical postbuckling strength for a given lateral maximum displacement,[4,5] and this fact can be used advantageously in the design of thermal structures. Thus it will be very useful to design engineers if simpler analytical methods are developed to predict the thermal postbuckling behavior of these structural elements.

Recently, the authors have proposed a simple intuitive method for predicting the thermal postbuckling behavior of uniform columns.[6] The method requires only the tension developed in columns with immovable ends because of large deformations and its critical load for given set of boundary conditions. Applicability of a similar method for evaluating the thermal postbuckling of thin isotropic square plates with immovable edges and different transverse boundary conditions is studied in the present Note. It is found that the present simple method is applicable to predicting the thermal postbuckling of square plates with particular sets of boundary condition, where the tensions developed in the plate, because of large deflections, are equal in the x and y directions (Fig. 1). Librescu et al.[7]

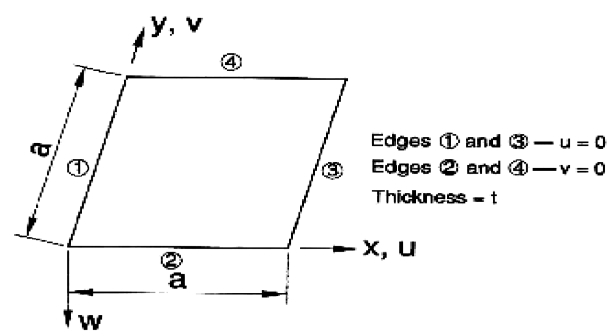

Fig. 1 Square plate with different lateral boundary condition sets on edges 1–4 subjected to uniform thermal load.

Received 1 August 2003; revision received 6 April 2004; accepted for publication 13 April 2004.

*Group Director, Structural Analysis and Testing Group; currently Professor, Department of Mechanical Engineering, Sreenidhi Institute of Science and Technology, Hyderabad 501 301, India; raohyd@hotmail.com.

†Head, Computational Structural Technology Division, Structural Analysis and Testing Group; kanakaraju_k@vssc.org.

and Librescu and Lin[8] have studied more general cases of plates and panels, considering the effects of tangential edge constraints. The applicability of the present method to more general cases[7,8] is to be studied, and the work is in progress.

It may be mentioned that the Rayleigh–Ritz method can be used to obtain the postbuckling behavior of simply supported plates only, as for this case it is possible to assume admissible expressions for the in-plane displacements u and v, whereas there is no logical way of assuming them for other sets of boundary conditions. The use of the finite element method for postbuckling problems involves the solution of an eigen value equation of large order (with the in-plane degrees of freedom included) through a parametric study with the lateral displacement parameter (δ/t) and requires large computer times.

Present Simple Method

Following Ref. 6, the ratio of the thermal postbuckling parameter to the thermal buckling parameter, λ_{PB}/λ_L, in the case of square plates subjected to a uniform thermal load, is given by

$$(\lambda_{PB}/\lambda_L) = 1 + \beta(\delta/t)^2 \tag{1}$$

where

$$\beta = (\bar{\lambda}_T/\lambda_L) \qquad \text{with} \qquad \lambda_T = \bar{\lambda}_T(\delta/t)^2 \tag{2}$$

The boundary configurations considered in the present study are such that $T_x = T_y = T$. It may be noted here that the thermal equivalent of the mechanical load N is given by $\alpha\Delta T Et/(1-\nu)$. As mentioned, the major task involved in the present method is the evaluation of the tension parameter λ_T developed in the square plate due to large deformations and the values of λ_L for the boundary conditions considered. The mechanical equivalent of the thermal-buckling-load parameter can be obtained from the standard books on the topic.[9,10]

Evaluation of λ_T

To evaluate λ_T developed due to large deflection, for square plates with different boundary conditions, the lateral displacement w in the x direction is assumed to be

$$w(x) = \delta F(x) \tag{3}$$

where δ is the central deflection and the function $F(x)$ satisfies the kinematic boundary conditions at the edges $x = 0$ and a and is so chosen that the deflection w at $x = a/2$ is unity.

With the edge at $x = 0$ immovable, the displacement at the edge at $x = a$ because of the tension T_x is

$$u_T = (T_x a/tE) \tag{4}$$

The displacement u_L (opposite in direction to u_T) because of large deflections is

$$u_L = \frac{1}{2}\int_0^a \left[\frac{\mathrm{d}}{\mathrm{d}x}w(x)\right]^2 \mathrm{d}x \tag{5}$$

Equating Eqs. (4) and (5), so that the edge at $x = a$ also becomes immovable, the in-plane tension T_x can be obtained as

$$T_x = \frac{Et}{2a}\int_0^a \left[\frac{\mathrm{d}}{\mathrm{d}x}w(x)\right]^2 \mathrm{d}x \tag{6}$$

and the tension parameter λ_{Tx} is defined as $T_x a^2/\pi^2 D$, to be consistent with the definition of λ_L.

For various expressions for $F(x)$ taken from Ref. 11, the tension parameters λ_{Tx} for different boundary condition sets at $x = 0$ and $x = a$ are given next:

1) For $F(x) = \sin(\pi x/a)$, for simply supported–simply supported plates (S–S),

$$\lambda_{Tx} = 3(1-\nu^2)(\delta/t)^2 \tag{7}$$

2) With $F(x) = \frac{1}{2}[1 - \cos(2\pi x/a)]$ for clamped–clamped plates (C–C),

$$\lambda_{Tx} = 3(1-\nu^2)(\delta/t)^2 \tag{8}$$

3) With $F(x) = 1/\sqrt{2}[\cos(\pi x/2a) - \cos(3\pi x/2a)]$ for clamped–simply supported plates (C–S),

$$\lambda_{Tx} = 15/4(1-\nu^2)(\delta/t)^2 \tag{9}$$

Similarly, the tension parameters developed due to large deflections in the y direction (λ_{Ty}) are also the same as those obtained in the x direction for S–S, C–C, and C–S conditions.

Evaluation of λ_{PB}/λ_L

The expressions for λ_{PB}/λ_L that give the postbuckling behavior of square plates with the boundary conditions 1, S–S–S–S; 2, C–C–C–C; 3, C–S–C–S; and 4, C–C–S–S are obtained in this section using Eq. (1). Referring to Fig. 1, C–S–C–S means edges 1 and 3 are clamped but edges 2 and 4 are simply supported. These four sets of boundary conditions develop uniform tensions $T_x = T_y = T$ in both x and y directions (which is consistent with the uniform biaxial mechanical load) because of a uniform rise of the temperature (ΔT) from the stress-free temperature state of the square plate.

With the tension parameter λ_T evaluated as in expressions (7–9) for different boundary conditions and with the known linear thermal load values, the values of β in Eq. (1) can be evaluated for the boundary conditions considered as 1) $\beta = 1.365$ for the S–S–S–S square plate ($\lambda_L = 2.0$), 2) $\beta = 0.5122$ for the C–C–C–C square plate ($\lambda_L = 5.33$), 3) $\beta = 0.7128$ for the C–S–C–S square plate ($\lambda_L = 3.83$), and 4) $\beta = 1.056$ for the C–C–S–S square plate ($\lambda_L = 3.2316$).

Numerical Results

From the values obtained for β based on the present simple method, the postbuckling results λ_{PB}/λ_L, are calculated for immovable square plates with S–S–S–S, C–C–C–C, C–S–C–S, and C–C–S–S boundary conditions, subjected to a uniform thermal load, for various values of δ/t and are presented in Table 1. The results available in Ref. 4 for the first three boundary conditions are also included in this table for the sake of comparison. It can be seen that

Table 1 Values of λ_{PB}/λ_L of a square plate for different sets of boundary conditions

	Boundary condition sets						
	S–S–S–S		C–C–C–C		C–S–C–S		C–C–S–S
δ/t	Present study	Ref. 4[a]	Present study	Ref. 4	Present study	Ref. 4	Present study
0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.2	1.0546	1.0610	1.0205	1.0195	1.0285	1.0270	1.0422
0.4	1.2184	1.2178	1.0819	1.0812	1.1140	1.1082	1.1690
0.6	1.4914	1.4878	1.1844	1.1819	1.2566	1.2480	1.3802
0.8	1.8736	1.8711	1.3278	1.3249	1.4562	1.4418	1.6758
1.0	2.3650	2.3676	1.5122	1.5068	1.7128	1.6898	2.0560

[a]Finite element method.

the present results agree very well with those obtained using the finite element method.[4]

Conclusions

The simple method originally developed by the authors to predict the postbuckling behavior of columns is shown to be applicable to predicting the thermal postbuckling behavior of square plates if the in-plane tensions developed in both the x and y directions of the plate are equal, which happens for certain sets of boundary conditions. It may be noted here that this is a successful preliminary study only and further work is certainly required to generalize the present study to the problem of thermal postbuckling of composite rectangular plates/panels with all possible sets of transverse boundary conditions, partially immovable edges, and other complicating effects.

References

[1]Thompson, J. M. T., and Hunt, G. W., *A General Theory of Elastic Stability*, Wiley, London, 1973.

[2]Dym, C. L., *Stability Theory and Its Application to Structural Mechanics*, Noordhoff International, Leyden, The Netherlands, 1974, Chap. 7.

[3]Rao, G. V., and Raju, K. K., "Thermal Post-Buckling of Columns," *AIAA Journal*, Vol. 22, No. 6, 1984, pp. 850, 851.

[4]Raju, K. K., and Rao, G. V., "Thermal Post-Buckling of a Square Plate Resting on an Elastic Foundation by Finite Element Method," *Computers and Structures*, Vol. 28, No. 2, 1988, pp. 195–199.

[5]Naidu, N. R., Raju, K. K., and Rao, G. V., "Post-Buckling of a Square Plate Resting on an Elastic Foundation Under Biaxial Compression," *Computers and Structures*, Vol. 37, No. 3, 1990, pp. 343–345.

[6]Rao, G. V., and Raju, K. K., "Thermal Postbuckling of Uniform Columns: A Simple Intuitive Method," *AIAA Journal*, Vol. 40, No. 10, 2002, pp. 2138–2140.

[7]Librescu, L., Lin, W., Nemeth, M. P., and Starnes, J. H., Jr., "Thermomechanical Postbuckling of Geometrically Imperfect Flat and Curved Panels Taking into Account Tangential Edge Constraints," *Journal of Thermal Stresses*, Vol. 18, 1995, pp. 465–482.

[8]Librescu, L., and Lin, W., "Vibration of Thermomechanically Loaded Flat and Curved Panels Taking into Account Geometric Imperfections and Tangential Edge Constraints," *International Journal of Solids and Structures*, Vol. 34, No. 17, 1997, pp. 2161–2181.

[9]Timoshenko, S. P., and Gere, J. M., *Theory of Elastic Stability*, McGraw–Hill, New York, 1961.

[10]Column Research Committee of Japan, *Handbook of Structural Stability*, Corona, Tokyo, 1971.

[11]Leissa, A. W., *Vibration of Plates*, NASA SP-160, 1969.

S. Saigal
Associate Editor